Preliminaries

- CNF – conjuctive normal form
    - *formula* = a conjuction of *clauses*
    - *clause* = a disjunction of *literals*
    - *literal* = (possibly negated) variables

- SAT-problem
    - 2-SAT – solvable in $O(n)$
    - 3-SAT – NP-hard
        - can solve general SAT

Local search - 2-SAT

- easy *heuristic* for hard problems:
    1. usually start with random solution
    2. apply small (local) changes, if beneficial
    - e.g. hillclimbing
        - e.g. 2-opt for TSP (cross quadruples)

- local search for SAT [Papadimitrou ’91]:
    1. start with random *valuation*
    2. repeat t times:
        - pick unsatisified clause
            - pick any literal and *flip*

- randomized algorithms: $P \leq ZPP \leq RP$
    - this will be in RP for the right t
        - always return NO for unsat.
        - *sometimes* return YES for sat.

- false negatives – how often?
    - but first: expected number of flips to sat.

- random walk:
    - assume sat. (unsat. is pointless)
    - let A = our valuation
    - let A^* = some satisfying valuation
    - walk on $Hamming(A, A^*)$

- Markov chain:
    - to right: $\geq \tfrac{1}{2}$
    - to left: $\leq \tfrac{1}{2}$
    - analysis → n^2

- Markov Inequality – if X is nonnegative, then:

  $$ P[X \geq a] \leq \frac{E[X]}{a} $$

- so if expected number of steps is n^2:

  $$ P[X \geq 2 E[X]] \leq \frac{E[X]}{2 E[X]} = \frac{1}{2}$$

- so for $t := 2 n^2$, it works with $p \geq \tfrac{1}{2}$ → RP

Local search - 3-SAT

- random walk – Markov chain:
    - to right: $\geq \tfrac{1}{3}$
    - to left: $\leq \tfrac{2}{3}$
    - no analysis → $O(2^n)$ – bad
        - optionally: Hongyu Sun on page 2

- what now?
    - bigger t doesn’t help
        - if we fail we might be very close to zero
    - instead: multiple tries from new random valuations
        - cf: random-restart hill-climbing