

# Homework Assignment 4

2-AIN-205, Spring 2025

Deadline: 26.05.2025, 22:00, google classroom

Before you start working on this homework assignment, read general instructions at the end of this document. You can write your solutions in Slovak or English. The solutions must be your work. Do not copy from others and do not attempt to find the solutions in literature or on the internet! Grade of 40 points will be considered as “full marks” for this assignment. The rest are bonus points.

1. [20 points] **Career advancement.** Unnamed company has  $n$  employees and career steps numbered  $1, 2, 3, \dots$ . An employee can be promoted from career step  $i$  to step  $i - 1$ . If promoted from career step 1, the employee moves to the board of directors which means that the employee gets to do nothing for the remainder of her/his life while being paid an exorbitant salary. Everybody wants to advance to the board of directors. Let  $a_i$  be the number of employees at career step  $i$ .

Promotions are awarded annually based on the following principle. The workers union proposes set  $S$  of employees for the promotion. The board of directors studies the proposal and chooses one of two options: either (a) promotes all employees in set  $S$ . or (b) fires employees in set  $S$  and promotes everybody else. The goal of the workers union is to get at least one employee to the board of directors. The board of directors, of course, wants to protect their untold riches, and does not want to promote new employees to the board.

In this game, both players (the union and the board) have full information (numbers of employees at individual career steps). Since there is no tie in the game, for every possible starting state  $a_1, a_2, \dots$ , one of the players must have a winning strategy.

- a) Assume that for some  $k$ ,  $a_k \geq 2^k$ . Show that in such case the workers union has a winning strategy.
  - b) Assume that the board's decisions are random, i.e. with probability  $1/2$  set  $S$  is promoted and with probability  $1/2$  set  $S$  is fired and everybody else is promoted. What is the probability that an employee starting at career step  $k$  makes it to the board of directors?
  - c) Show that expected number of employees that will make it to the board of directors is in such case  $\sum_k a_k/2^k$ .
  - d) (**bonus**) Show that if at the beginning  $\sum_k a_k/2^k < 1$  the board has a winning strategy.
2. [20 points] **From Monte Carlo to Las Vegas.** Assume that we have a two-sided Monte Carlo algorithm with running time  $T(n)$  which returns erroneous answer with probability  $p$ . At the same time, the algorithm also provides an additional information which can be used to further verify the validity of the answer in time  $T'(n)$ . Show that in such case it is possible to create Las Vegas algorithm with running time  $(T(n) + T'(n))/(1 - p)$ .
  3. [20 points] **Programming task** (see general instructions).

Submission link: <https://judge.ksp.sk/public/submit/7c44746f7a13bf99af5bb6047b83f158/>

On the input there are  $n$  strings composed of characters 0, 1. Your task is to find out whether they have a common subsequence of length at least  $k$ . (Note that the subsequence does not need to be continuous, i.e. strings 1111 and 110011 have common subsequence 1111.)

**Input specification:** On the first input line there are two numbers  $n$  and  $k$ . Each of the next  $n$  lines contains one string.

**Output specification:** If given strings have a common subsequence of length  $k$ , output YES, otherwise output NO.

**Restrictions and grading:** To obtain full number of points, your program has to give a correct answer for inputs where  $n \leq 10$ ,  $k \leq 18$  and the length of each string is at most 1000.

**Exmple input:**

3 3  
01010  
00000  
10100

**Example output:**

YES  
Note: The strings contain a common subsequence 000.

## General Instructions

**Theoretical tasks.** Submit answers to theoretical tasks in one pdf file per task in google classroom (your answer can be typeset or scanned, but please make sure that everything is legible and easy to read). No late submissions are allowed.

Write your solutions so that they contain all information necessary to easily understand them, but at the same time try to aim for brevity. Prove all claims, including in the cases when it is not explicitly written in the problem statement.

If the task is about solving an algorithmic problem, submit the best algorithm you can design. The first criterion is that the algorithm *is correct*, the second criterion is the *running time and memory complexity*. Correct and slow algorithm is worth more points than fast incorrect algorithm. Inefficient but correct algorithms will always receive at least 50% of points. In your solution, you should:

- Describe the main idea of the algorithm.
- Write a pseudocode for your algorithm.
- Prove the correctness of your algorithm.
- Analyse its running time and memory complexity.

**Programming tasks.** You need to submit a functional program, no written part is required. Your solutions will be evaluated immediately on several inputs and you will learn number of points (there are several sets of inputs and you only gain points if you solve correctly all inputs in a particular set). You can submit a solution several times, we will only take into account the last submitted solution before the deadline. To submit your solution, please use “Univerzita Komenského” login on <https://judge.ksp.sk/> system. You can find instructions on how to submit your programming task solutions at <https://judge.ksp.sk/docs/>.