Strong Lagrange Duality (with equality constraints)

Let $f : R^n \to R$ a $g : R^n \to R^m$ are **convex** functions,

$h : R^n \to R^k$ is **affine**

X is a closed convex set over R^n

there exists $\hat{x} \in X : g(\hat{x}) < 0$ and $h(\hat{x}) = 0$.

Then:

if x^* **minimizes** $f(x)$ subject to $g(x) \leq 0$, $h(x) = 0$, $x \in X$

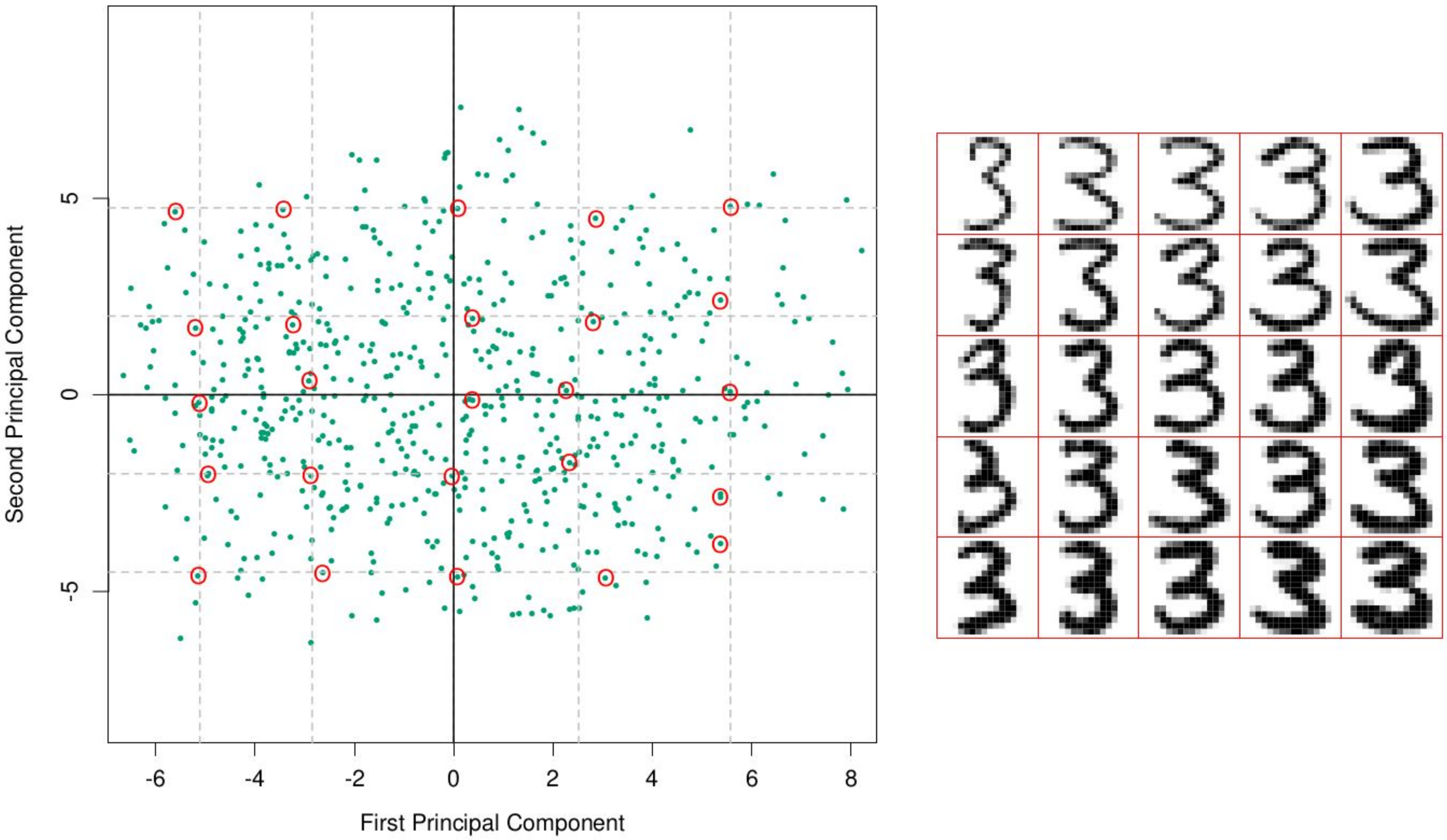
$\nwarrow$ primal program

$(\lambda_g^*, \lambda_h^*)$ **maximizes** $L(\lambda_g, \lambda_h)$ subject to $\lambda_g \geq 0$ $\quad \leftarrow$ dual program

then $f(x^*) = L(\lambda_g^*, \lambda_h^*)$

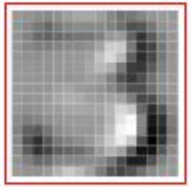
$$L(\lambda_g, \lambda_h) = \min_{x \in X} \underbrace{\{f(x) + \langle \lambda_g, g(x) \rangle + \langle \lambda_h, h(x) \rangle\}}_{\ell(\lambda_g, \lambda_h, x)}$$

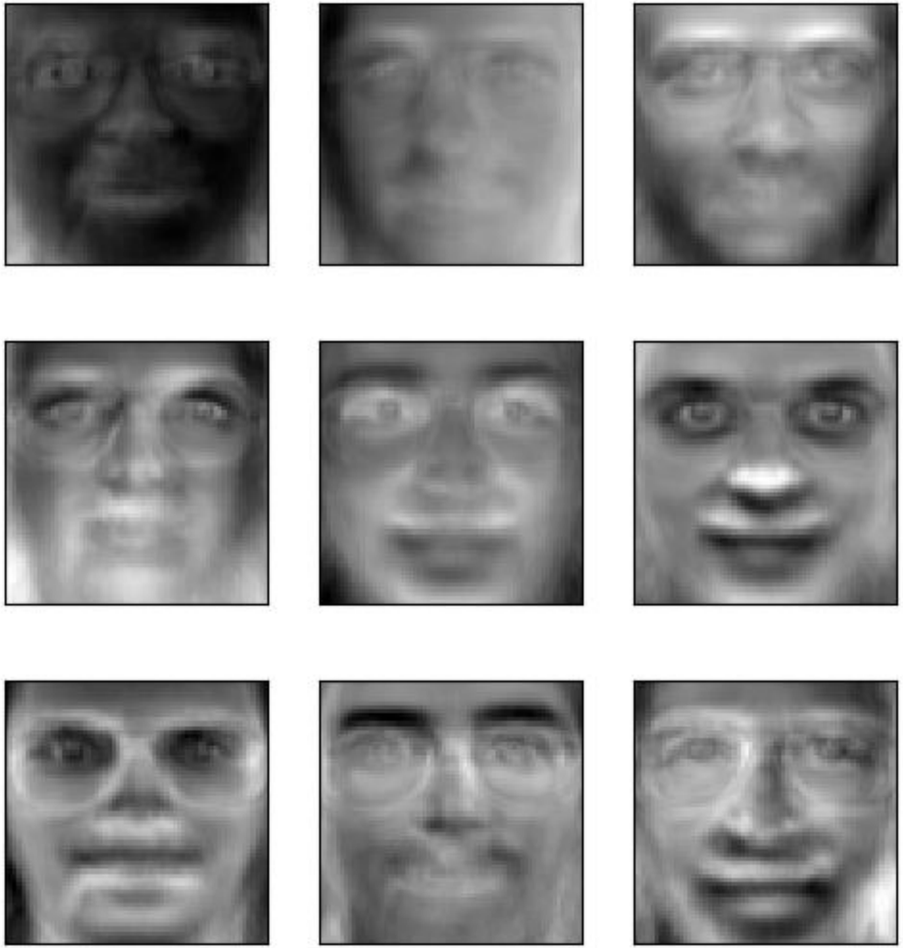
vectors λ_g, λ_h: so called Lagrange multipliers



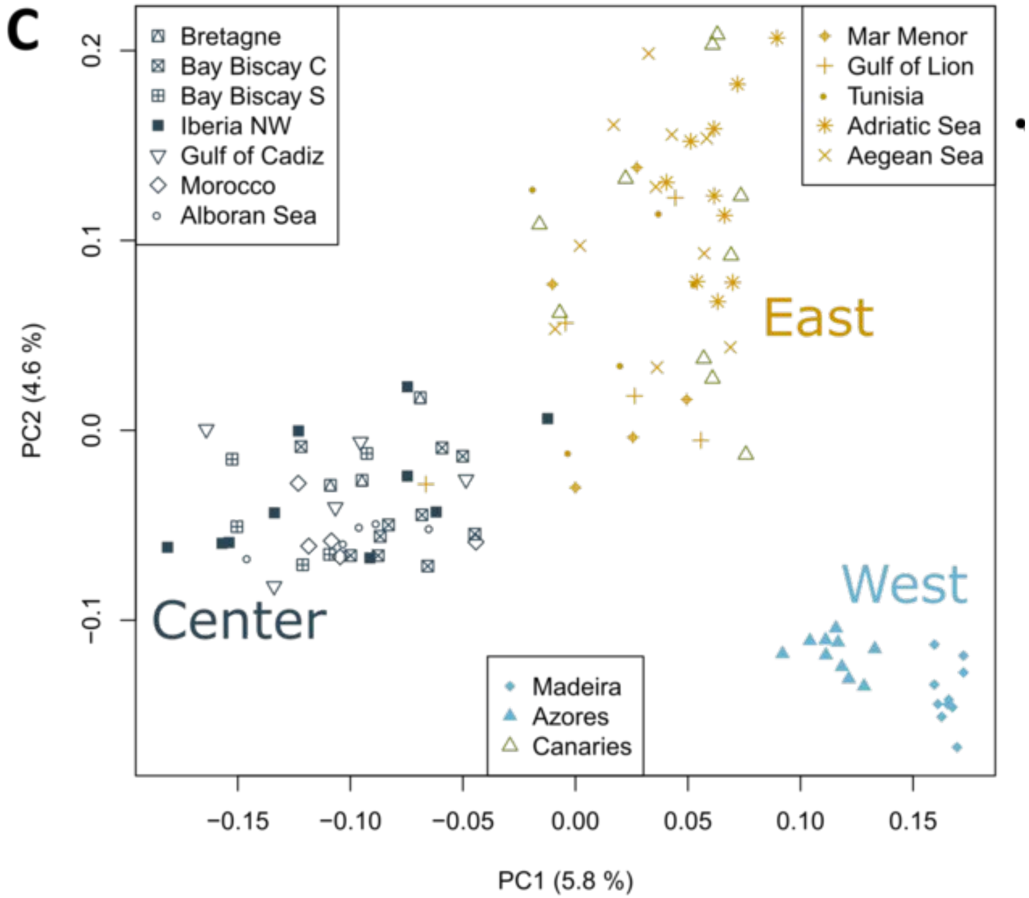
Second Principal Component
First Principal Component
5
0
-5
-6
-4
-2
0
2
4
6
8

$$\hat{f}(\lambda) = \bar{x} + \lambda_1 v_1 + \lambda_2 v_2$$

$=$  $+ \lambda_1 \cdot$  $+ \lambda_2 \cdot$  .



PARMESANE
CONSERVERIE
la belle-iloise
Crème de Saumon
A L'ESTRAGON
CONSERVERIE
la belle-iloise
Mousse
SARDINES
à l'huile
d'Arachide
CONSERVERIE
la belle-iloise
Sardines
à la tapenade
CONSERVERIE
la belle-iloise
matiz
WILD
SARDINES
IN SPANISH OLIVE OIL
Wild caught in the Eastern Atlantic
2 grams of Omega-3s per serving and high in Calcium
NET WT: 120g (4.2 oz)



C
Bretagne
Bay Biscay C
Bay Biscay S
Iberia NW
Gulf of Cadiz
Morocco
Alboran Sea
Mar Menor
Gulf of Lion
Tunisia
Adriatic Sea
Aegean Sea
Madeira
Azores
Canaries
East
West
Center
PC2 (4.6 %)
PC1 (5.8 %)
0.2
0.1
0.0
−0.1
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15