

Vyšetřovanie priebehu viacrozmerých funkcií

- používame **parciálne derivácie**
- zderivujeme podľa jednej premennej
ostatné premenné považujeme za konštanty:

$$\frac{\partial f(a, b)}{\partial a} = \lim_{a \rightarrow x} \frac{f(x, b) - f(a, b)}{x - a}$$

- **nutná podmienka:** ak f má v bode lokálny extrém, potom **všetky** parciálne derivácie musia byť nulové

Násobenie matíc

$$A_{m \times n} \cdot B_{n \times k} = C_{m \times k}$$

$$C_{ij} = \sum_{u=1}^n A_{iu} B_{uj}$$

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}_{3 \times 3} \cdot \begin{bmatrix} 2 & 9 \\ 0 & -2 \\ -1 & 2 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 1 & 5 \\ 0 & 11 \\ 0 & 20 \end{bmatrix}_{3 \times 2}$$

Inverzné matice

$$A_{m \times n} \cdot B_{n \times k} = C_{m \times k}$$

$$C_{ij} = \sum_{u=1}^n A_{iu} B_{uj}$$

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}_{3 \times 3} \cdot \underbrace{\begin{bmatrix} 2 & 9 & -5 \\ 0 & -2 & 1 \\ -1 & -3 & 2 \end{bmatrix}_{3 \times 3}}_{\text{inverzná matica}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}}_{\text{jednotková matica}}$$

$$A \cdot A^{-1} = I$$

$$(AB)^{-1} = B^{-1} \cdot A^{-1}$$

Transponované matice

$$A_{n \times m} = (a_{ij})_{n \times m}$$

$$(A^T)_{m \times n} = (a_{ji})_{m \times n}$$

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}_{3 \times 3} \cdot \begin{bmatrix} 2 & 9 \\ 0 & -2 \\ -1 & 2 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 1 & 5 \\ 0 & 11 \\ 0 & 20 \end{bmatrix}_{3 \times 2}$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 9 & -2 & -3 \end{bmatrix}_{2 \times 3} \cdot \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 11 & 20 \end{bmatrix}_{2 \times 3}$$

$$(A \cdot B)^T = B^T \cdot A^T$$

Gradient

Majme funkciju $f : R_{m \times n} \rightarrow R$

$$\nabla_A f(A) = \begin{bmatrix} \frac{\partial f(A)}{\partial a_{11}} & \frac{\partial f(A)}{\partial a_{12}} & \cdots & \frac{\partial f(A)}{\partial a_{1n}} \\ \frac{\partial f(A)}{\partial a_{21}} & \frac{\partial f(A)}{\partial a_{22}} & \cdots & \frac{\partial f(A)}{\partial a_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f(A)}{\partial a_{m1}} & \frac{\partial f(A)}{\partial a_{m2}} & \cdots & \frac{\partial f(A)}{\partial a_{mn}} \end{bmatrix}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(AB)^T = B^T A^T$$

Vektor x : matica s jedným **stĺpcem**

$$\text{Skalárny súčin: } \langle x, y \rangle = \sum_{i=1}^n x_i y_i = x^T y$$

$$\text{Stopa matice: } \text{tr} A_{n \times n} = \sum_{i=1}^n A_{ii}$$

$$\text{tr}(A + B) = \text{tr} A + \text{tr} B$$

$$\text{tr} A^T = \text{tr} A$$

$$\nabla_A \text{tr}(BA) = B^T$$

$$\nabla_A \text{tr}(A^T B A C) = B^T A C^T + B A C$$