Strong Lagrange Duality

Let $f : R^n \to R$ a $g : R^n \to R^m$ are **convex** functions,

X is a closed convex set over R^n

there exists $\hat{x} \in X : g(\hat{x}) < 0$.

Then:

if x^* **minimizes** $f(x)$ subject to $g(x) \leq 0$, $x \in X$

$\nwarrow$ primal program

λ^* **maximizes** $L(\lambda)$ subject to $\lambda \geq 0$ $\leftarrow$ dual program

then $f(x^*) = L(\lambda^*)$

$$L(\lambda) = \min_{x \in X} \underbrace{\{f(x) + \langle \lambda, g(x) \rangle\}}_{\ell(\lambda, x)}$$

vector λ: so called Lagrange multipliers

Strong Lagrange Duality (with equality constraints)

Let $f : R^n \to R$ a $g : R^n \to R^m$ are **convex** functions,

$h : R^n \to R^k$ is **affine**

X is a closed convex set over R^n

there exists $\hat{x} \in X : g(\hat{x}) < 0$ and $h(\hat{x}) = 0$.

Then:

if x^* **minimizes** $f(x)$ subject to $g(x) \leq 0$, $h(x) = 0$, $x \in X$

$\nwarrow$ primal program

$(\lambda_g^*, \lambda_h^*)$ **maximizes** $L(\lambda_g, \lambda_h)$ subject to $\lambda_g \geq 0$ $\leftarrow$ dual program

then $f(x^*) = L(\lambda_g^*, \lambda_h^*)$

$$L(\lambda_g, \lambda_h) = \min_{x \in X} \underbrace{\{f(x) + \langle \lambda_g, g(x) \rangle + \langle \lambda_h, h(x) \rangle\}}_{\ell(\lambda_g, \lambda_h, x)}$$

vectors λ_g, λ_h: so called Lagrange multipliers