

# Theoretical homework 1

2-AIN-150, Winter 2025

Deadline: 27.10.2025, 23:59, Google classroom

Before you start solving the homework, please read the general instruction at the end of the document. Submitted solutions should be your own. Do not copy and do not try to find solution in literature or over the internet.

## Theory of ML

Consider problem of regression over set of hypotheses  $H = \{h_b(x) = 2x + b\}$ .

- a) Describe algorithm, which takes given training data  $(x^{(1)}, y^{(1)}), \dots, (x^{(n)}, y^{(n)})$  and selects hypothesis, which minimizes error function:  $E(b) = \sum_{i=1}^n (h_b(x^{(i)}) - y^{(i)})^2$ .
- b) If data are generated using distribution  $P_{x,y}$  defined as:
- distribution of  $x$  is uniform over interval  $[0, 100]$ .
  - for given  $x$  is  $\Pr(y = 2x + 7|x) = 0.4$  and  $\Pr(y = 2x - 5|x) = 0.6$  (other values of  $y$  with given  $x$  are not possible).

What is the best possible test error for hypothesis set  $H$  (i.e. test for best hypothesis out of  $H$ ) if we assume, that data are independent samples from  $P_{x,y}$ .

- c) For distribution  $P_{x,y}$  from part b) and  $n = 1$  calculate expected training and test error (i.e. we generate one sample, we train and from this process we calculate expected errors).
- d) For distribution  $P_{x,y}$  from part b) and general  $n$  calculate expected training and test error. What is the relationship of these errors with optimal test error if  $n \rightarrow \infty$ ? How would you set good size of training sample based on your results?

## General instructions

Submit a solution in PDF format via Google classroom.