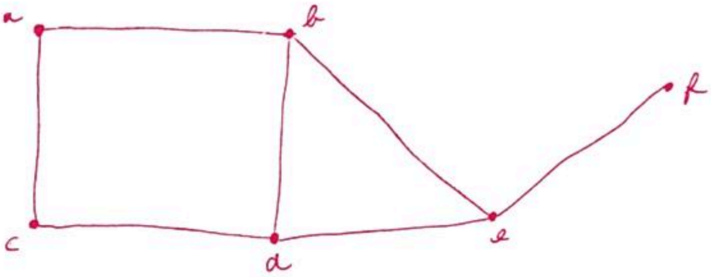
MAXIMUM MATCHING

$G = (V, E)$



MATCHING $M_1 = \{ (b, e) \}$

↳ NO 2 EDGES SHARE A COMMON VERTEX

MAXIMAL MATCHING $M_2 = \{ (b, e), (a, c) \}$

↳ IMPOSSIBLE TO ADD ANOTHER EDGE TO MATCHING

MAXIMUM MATCHING $M_3 = \{ (a, c), (b, d), (e, f) \}$

↳ MATCHING WITH LARGEST POSSIBLE NUMBER OF EDGES

EXACT SOLUTION

↳ BLOSSOM ALGORITHM - TIME $O(|E||V|^2)$

2-APX ALGORITHM - TIME $(|E| + |V|)$

↳ $M = \{\}$

WHILE $E \neq \emptyset$:

RANDOMLY SELECT $(u, v) \in E$

$M = M \cup \{(u, v)\}$

DELETE $\forall$ EDGES INCIDENT TO u AND v

RETURN M

PROOF:

M = MAXIMUM MATCHING

M' = 2-APX ALG MATCHING

$e \in M' \Rightarrow e \in M \lor e \notin M$

↳ $e \in M \Rightarrow$ OK

↳ $e \notin M \Rightarrow e = (u, v)$, BOTH u, v MATCHED TO OTHER VERTEX IN M

$|M_e| \leq 2$

$$|M| \leq \sum_{e \in M'} |M_e| \leq 2|M'|$$

RECTANGLES

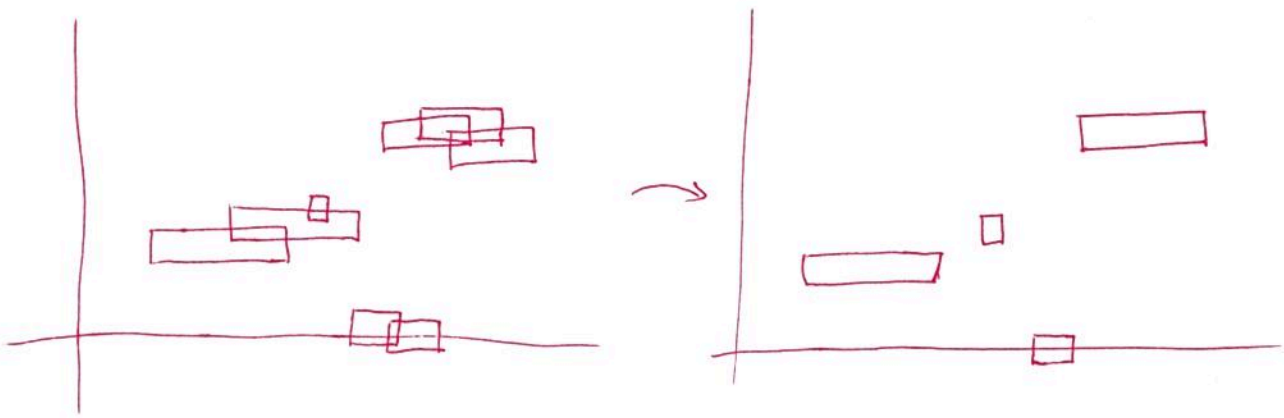
n RECTANGLES

$r_i = (x, y, \text{WIDTH}, \text{HEIGHT} = 1)$

SELECT SUBSET OF RECTANGLES

↳ NOT OVERLAYING

↳ MAX TOTAL AREA



2-APX ALG ?

↳ PROBLEM ON X-AXIS - DYN. PROG - WEIGHTED ACTIVITY SELECTION PROBLEM

↳ SOLVE FOR FLOOR(Y) % 2 == 0

MINIMUM MAKESPAN SCHEDULING

n JOBS

PROCESSING TIMES $p_1, p_2, \dots, p_n$

m MACHINES

GOAL: ASSIGN JOBS TO MACHINES SO THAT THE COMPLETION TIME IS MINIMIZED (MAKESPAN)

2-APX ALG:

FOR EACH JOB:

ASSIGN TO A MACHINE WITH LEAST AMOUNT OF WORK SO FAR

PROOF:

M_i - LAST MACHINE TO FINISH

p_j - LAST JOB SCHEDULED ON M_i

ALG ASSIGNS A JOB TO THE LEAST LOADED MACHINE

$\Rightarrow \forall$ MACHINES BUSY UNTIL p_j.START

$$\text{START}.p_j \leq \frac{1}{m}\sum_i p_i \leq LB = \max\left\{\frac{1}{m}\sum_i p_i, \max_i\{p_i\}\right\} \leq OPT$$

$p_j \leq OPT$

$\Rightarrow p_j + p_j.\text{START} \leq 2 \cdot OPT$ □

WORST CASE:

m^2 JOBS OF TIME 1

1 JOB OF TIME m

$\to$ ALG $\to 2m$

OPT $\to m+1$