EADŠ - cvičenie 1 - zložitosti

21. septembra 2023

Opakovanie - zložitosti

Definition

Function $f(n)$ is in $O(g(n))$ iff there exist $c > 0$ and $n_0 > 0$ such that: $(\forall n > n_0)(0 \leq f(n) \leq cg(n))$

Trieda	Intuitívne	Limita
$f(n) \in O(g(n))$	$f(n) \leq g(n)$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq \infty$
$f(n) \in \Omega(g(n))$	$f(n) \geq g(n)$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0$
$f(n) \in \Theta(g(n))$	$f(n) \approx g(n)$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0, \infty$

Opakovanie - určovanie zložitosti

- zanedbávame konštanty:
  $O(c \cdot f(n)) = O(f(n))$
  (napr. $O(3n) = O(n)$)
- berieme najväčšiu funkciu
  $O(f(n) + g(n)) = O(\max(f(n), g(n)))$
  (napr. $O(n^5 + n^2 + n) = O(n^5)$)
- poradie zložitostí:
  $\forall a > 1 : O(1) \in O(\log^a n) \in O(n^a) \in O(a^n) \in O(n!) \in O(n^n)$
  Intuitívne:
  $\forall a > 1 : 1 \leq \log^a n \leq n^a \leq a^n \leq n! \leq n^n$

Opakovanie - čo je treba vedieť

- určovanie zložitosti z kódu
- asymptotické porovnávanie dvoch funkcií (O, Ω, Θ)

Príklad

```
x = 0
for i in range(2*n+15):
    x = x + i
```

Príklad

```
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for i in range(2*n+15):
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```

$f(n) \approx$

Tvrdenie	platí?
$f(n) = 2n \in O(n^2)$	
$f(n) = 2n \in \Omega(\log n)$	
$f(n) = 2n \in \Theta(\log n)$	

Príklad

```
x = 0
for i in range(2*n+15):
    x = x + i
```

$f(n) \approx 2n$

Tvrdenie	platí?
$f(n) = 2n \in O(n^2)$	
$f(n) = 2n \in \Omega(\log n)$	
$f(n) = 2n \in \Theta(\log n)$	

Príklad

```
x = 0
for i in range(2*n+15):
    x = x + i
```

$f(n) \approx 2n$

Tvrdenie	platí?
$f(n) = 2n \in O(n^2)$	✓
$f(n) = 2n \in \Omega(\log n)$	✓
$f(n) = 2n \in \Theta(\log n)$	✗

A

```
x = 0
for i in range(4 * n, 0, -1):
    x = x + 2 * i
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(\sqrt{n})$	
$f(n) \in \Omega(\sqrt{n})$	
$f(n) \in \Theta(\sqrt{n})$	

A

```
x = 0
for i in range(4 * n, 0, -1):
    x = x + 2 * i
```

$f(n) \approx 4n$

Tvrdenie	platí?
$f(n) \in O(\sqrt{n})$	
$f(n) \in \Omega(\sqrt{n})$	
$f(n) \in \Theta(\sqrt{n})$	

A

```
x = 0
for i in range(4 * n, 0, -1):
    x = x + 2 * i
```

$f(n) \approx 4n$

Tvrdenie	platí?
$f(n) \in O(\sqrt{n})$	✗
$f(n) \in \Omega(\sqrt{n})$	✓
$f(n) \in \Theta(\sqrt{n})$	✗

B

```
z = 0
x = 0
i = 1
while i <= n:
    z += 5
    x *= 2
    i *= 3
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(5\sqrt{n})$	
$f(n) \in \Omega(5\sqrt{n})$	
$f(n) \in \Theta(5\sqrt{n})$	

B

```
z = 0
x = 0
i = 1
while i <= n:
    z += 5
    x *= 2
    i *= 3
```

$f(n) \approx \log_3 n$

Tvrdenie	platí?
$f(n) \in O(5\sqrt{n})$	
$f(n) \in \Omega(5\sqrt{n})$	
$f(n) \in \Theta(5\sqrt{n})$	

B

```
z = 0
x = 0
i = 1
while i <= n:
    z += 5
    x *= 2
    i *= 3
```

$f(n) \approx \log_3 n$

Tvrdenie	platí?
$f(n) \in O(5\sqrt{n})$	✓
$f(n) \in \Omega(5\sqrt{n})$	✗
$f(n) \in \Theta(5\sqrt{n})$	✗

C

```
y = 0
j = 1
while j*j <= n:
    y += 1
    j += 1
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(n^{0.5} + \log n)$	
$f(n) \in \Omega(n^{0.5} + \log n)$	
$f(n) \in \Theta(n^{0.5} + \log n)$	

C

```
y = 0
j = 1
while j*j <= n:
    y += 1
    j += 1
```

$f(n) \approx \sqrt{n}$

Tvrdenie	platí?
$f(n) \in O(n^{0.5} + \log n)$	
$f(n) \in \Omega(n^{0.5} + \log n)$	
$f(n) \in \Theta(n^{0.5} + \log n)$	

C

```
y = 0
j = 1
while j*j <= n:
    y += 1
    j += 1
```

$f(n) \approx \sqrt{n}$

Tvrdenie	platí?
$f(n) \in O(n^{0.5} + \log n)$	✓
$f(n) \in \Omega(n^{0.5} + \log n)$	✓
$f(n) \in \Theta(n^{0.5} + \log n)$	✓

D

```
b = 0
i = n
for i in range(n, 0, -1):
    for j in range(0, i, 1):
        b += 5
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(n^{1.98} + \sqrt{n})$	
$f(n) \in \Omega(n^{1.98} + \sqrt{n})$	
$f(n) \in \Theta(n^{1.98} + \sqrt{n})$	

D

```
b = 0
i = n
for i in range(n, 0, -1):
    for j in range(0, i, 1):
        b += 5
```

$f(n) \approx n^2$

Tvrdenie	platí?
$f(n) \in O(n^{1.98} + \sqrt{n})$	
$f(n) \in \Omega(n^{1.98} + \sqrt{n})$	
$f(n) \in \Theta(n^{1.98} + \sqrt{n})$	

D

```
b = 0
i = n
for i in range(n, 0, -1):
    for j in range(0, i, 1):
        b += 5
```

$f(n) \approx n^2$

Tvrdenie	platí?
$f(n) \in O(n^{1.98} + \sqrt{n})$	✗
$f(n) \in \Omega(n^{1.98} + \sqrt{n})$	✓
$f(n) \in \Theta(n^{1.98} + \sqrt{n})$	✗

E

```
y = 0
for j in range(0, 2*n + 1, 2):
    y += j

s = 0
for i in range(1, j + 1, 1):
    s += 1
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(2n \log n)$	
$f(n) \in \Omega(2n \log n)$	
$f(n) \in \Theta(2n \log n)$	

E

```
y = 0
for j in range(0, 2*n + 1, 2):
    y += j

s = 0
for i in range(1, j + 1, 1):
    s += 1
```

$f(n) \approx n$

Tvrdenie	platí?
$f(n) \in O(2n \log n)$	
$f(n) \in \Omega(2n \log n)$	
$f(n) \in \Theta(2n \log n)$	

E

```
y = 0
for j in range(0, 2*n + 1, 2):
    y += j

s = 0
for i in range(1, j + 1, 1):
    s += 1
```

$f(n) \approx n$

Tvrdenie	platí?
$f(n) \in O(2n \log n)$	✓
$f(n) \in \Omega(2n \log n)$	✗
$f(n) \in \Theta(2n \log n)$	✗

F

```
b = 0
for i in range(1, n+1, 1):
    for j in range(1, (i*n)+1, 1):
        b += 5
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(500\,000 \cdot n^{2.99})$	
$f(n) \in \Omega(500\,000 \cdot n^{2.99})$	
$f(n) \in \Theta(500\,000 \cdot n^{2.99})$	

F

```
b = 0
for i in range(1, n+1, 1):
    for j in range(1, (i*n)+1, 1):
        b += 5
```

$f(n) \approx n^3$

Tvrdenie	platí?
$f(n) \in O(500\,000 \cdot n^{2.99})$	
$f(n) \in \Omega(500\,000 \cdot n^{2.99})$	
$f(n) \in \Theta(500\,000 \cdot n^{2.99})$	

F

```
b = 0
for i in range(1, n+1, 1):
    for j in range(1, (i*n)+1, 1):
        b += 5
```

$f(n) \approx n^3$

Tvrdenie	platí?
$f(n) \in O(500\,000 \cdot n^{2.99})$	✗
$f(n) \in \Omega(500\,000 \cdot n^{2.99})$	✓
$f(n) \in \Theta(500\,000 \cdot n^{2.99})$	✗

G

```
x = 0
i = 1
while i <= n:
    if i % 2 != 0:
        for j in range(0, i, 1):
            x += 1
    i *= 3
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(n! + n^{200})$	
$f(n) \in \Omega(n! + n^{200})$	
$f(n) \in \Theta(n! + n^{200})$	

G

```
x = 0
i = 1
while i <= n:
    if i % 2 != 0:
        for j in range(0, i, 1):
            x += 1
    i *= 3
```

$f(n) \approx n$

Tvrdenie	platí?
$f(n) \in O(n! + n^{200})$	
$f(n) \in \Omega(n! + n^{200})$	
$f(n) \in \Theta(n! + n^{200})$	

G

```
x = 0
i = 1
while i <= n:
    if i % 2 != 0:
        for j in range(0, i, 1):
            x += 1
    i *= 3
```

$f(n) \approx n$

Tvrdenie	platí?
$f(n) \in O(n! + n^{200})$	✓
$f(n) \in \Omega(n! + n^{200})$	✗
$f(n) \in \Theta(n! + n^{200})$	✗

H

```
t = 0
for i in range(1, n+1, 1):
    j = 0
    while j*j < 4*n:
        k = 1
        while k*k <= 9*n:
            t += 1
            k += 1
        j += 1
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(n \cdot logn)$	
$f(n) \in \Omega(n \cdot logn)$	
$f(n) \in \Theta(n \cdot logn)$	

H

```
t = 0
for i in range(1, n+1, 1):
    j = 0
    while j*j < 4*n:
        k = 1
        while k*k <= 9*n:
            t += 1
            k += 1
        j += 1
```

$f(n) \approx n^2$

Tvrdenie	platí?
$f(n) \in O(n \cdot logn)$	
$f(n) \in \Omega(n \cdot logn)$	
$f(n) \in \Theta(n \cdot logn)$	

H

```
t = 0
for i in range(1, n+1, 1):
    j = 0
    while j*j < 4*n:
        k = 1
        while k*k <= 9*n:
            t += 1
            k += 1
        j += 1
```

$f(n) \approx n^2$

Tvrdenie	platí?
$f(n) \in O(n \cdot logn)$	✗
$f(n) \in \Omega(n \cdot logn)$	✓
$f(n) \in \Theta(n \cdot logn)$	✗

```
a = 0
k = n*n
while k > 1:
    for j in range(0, n*n, 1):
        a += 1
    k /= 2
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(n^{2.0000001} + \log n)$	
$f(n) \in \Omega(n^{2.0000001} + \log n)$	
$f(n) \in \Theta(n^{2.0000001} + \log n)$	

```
a = 0
k = n*n
while k > 1:
    for j in range(0, n*n, 1):
        a += 1
    k /= 2
```

$f(n) \approx n^2 \cdot logn$

Tvrdenie	platí?
$f(n) \in O(n^{2.0000001} + \log n)$	
$f(n) \in \Omega(n^{2.0000001} + \log n)$	
$f(n) \in \Theta(n^{2.0000001} + \log n)$	

```
a = 0
k = n*n
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    for j in range(0, n*n, 1):
        a += 1
    k /= 2
```

$f(n) \approx n^2 \cdot logn$

Tvrdenie	platí?
$f(n) \in O(n^{2.0000001} + \log n)$	✓
$f(n) \in \Omega(n^{2.0000001} + \log n)$	✗
$f(n) \in \Theta(n^{2.0000001} + \log n)$	✗

J

```
i = 0
j = 0
y = 0
s = 0
for j in range(1, n+1, 1):
    y += j
    for i in range(1, y+1, 1):
        s += 1
```

J

```
i = 0
j = 0
y = 0
s = 0
for j in range(1, n+1, 1):
    y += j
    for i in range(1, y+1, 1):
        s += 1
```

$f(n) \approx n^3$

Tvrdenie	platí?
$f(n) \in O(n^{2.500001} \cdot \sqrt{n})$	
$f(n) \in \Omega(n^{2.500001} \cdot \sqrt{n})$	
$f(n) \in \Theta(n^{2.500001} \cdot \sqrt{n})$	

J

```
i = 0
j = 0
y = 0
s = 0
for j in range(1, n+1, 1):
    y += j
    for i in range(1, y+1, 1):
        s += 1
```

$f(n) \approx n^3$

Tvrdenie	platí?
$f(n) \in O(n^{2.500001} \cdot \sqrt{n})$	✓
$f(n) \in \Omega(n^{2.500001} \cdot \sqrt{n})$	✗
$f(n) \in \Theta(n^{2.500001} \cdot \sqrt{n})$	✗

K

```
i = 1
z = 0
while z < n*(n+1)/2:
    z += i
    i += 1
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(\sqrt{n})$	
$f(n) \in \Omega(\sqrt{n})$	
$f(n) \in \Theta(\sqrt{n})$	

K

```
i = 1
z = 0
while z < n*(n+1)/2:
    z += i
    i += 1
```

$f(n) \approx n$

Tvrdenie	platí?
$f(n) \in O(\sqrt{n})$	
$f(n) \in \Omega(\sqrt{n})$	
$f(n) \in \Theta(\sqrt{n})$	

K

```
i = 1
z = 0
while z < n*(n+1)/2:
    z += i
    i += 1
```

$f(n) \approx n$

Tvrdenie	platí?
$f(n) \in O(\sqrt{n})$	✗
$f(n) \in \Omega(\sqrt{n})$	✓
$f(n) \in \Theta(\sqrt{n})$	✗

L

```
a = 0
k = n*n*n
while k > 1:
    for j in range(0, k, 1):
        a -= 1
    k /= 2
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(n^2 \cdot n \log n)$	
$f(n) \in \Omega(n^2 \cdot n \log n)$	
$f(n) \in \Theta(n^2 \cdot n \log n)$	

L

```
a = 0
k = n*n*n
while k > 1:
    for j in range(0, k, 1):
        a -= 1
    k /= 2
```

$f(n) \approx n^3$

Tvrdenie	platí?
$f(n) \in O(n^2 \cdot n \log n)$	
$f(n) \in \Omega(n^2 \cdot n \log n)$	
$f(n) \in \Theta(n^2 \cdot n \log n)$	

L

```
a = 0
k = n*n*n
while k > 1:
    for j in range(0, k, 1):
        a -= 1
    k /= 2
```

$f(n) \approx n^3$

Tvrdenie	platí?
$f(n) \in O(n^2 \cdot n \log n)$	✓
$f(n) \in \Omega(n^2 \cdot n \log n)$	✗
$f(n) \in \Theta(n^2 \cdot n \log n)$	✗

M

```
for i in range(1, n, 1):
    j = 0
    while j < n:
        x += 1
        j += i
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(n^{1.1})$	
$f(n) \in \Omega(n^{1.1})$	
$f(n) \in \Theta(n^{1.1})$	

M

```
for i in range(1, n, 1):
    j = 0
    while j < n:
        x += 1
        j += i
```

$f(n) \approx n \log n$

Tvrdenie	platí?
$f(n) \in O(n^{1.1})$	
$f(n) \in \Omega(n^{1.1})$	
$f(n) \in \Theta(n^{1.1})$	

M

```
for i in range(1, n, 1):
    j = 0
    while j < n:
        x += 1
        j += i
```

$f(n) \approx n \log n$

Tvrdenie	platí?
$f(n) \in O(n^{1.1})$	✓
$f(n) \in \Omega(n^{1.1})$	✗
$f(n) \in \Theta(n^{1.1})$	✗

N - KMP

```
m = n // 20  # kmp preparation
A, B = "A"*n, "A"*m + "$"
kmp = [0] + list(range(m))  # kmp[0] = 0, kmp[i] < i
```

N - KMP

```
m = n // 20  # kmp preparation
A, B = "A"*n, "A"*m + "$"
kmp = [0] + list(range(m))  # kmp[0] = 0, kmp[i] < i


cur = 0
for i in range(n):
    while cur != 0 and A[i] != B[cur]:
        cur = kmp[cur]
    if A[i] == B[cur]:
        cur += 1
```

$f(n) \approx$

Tvrdenie	platí?
$f(n) \in O(\sqrt{n} \cdot \sqrt{n})$	
$f(n) \in \Omega(\sqrt{n} \cdot \sqrt{n})$	
$f(n) \in \Theta(\sqrt{n} \cdot \sqrt{n})$	

N - KMP

```
m = n // 20  # kmp preparation
A, B = "A"*n, "A"*m + "$"
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cur = 0
for i in range(n):
    while cur != 0 and A[i] != B[cur]:
        cur = kmp[cur]
    if A[i] == B[cur]:
        cur += 1
```

$f(n) \approx n$

Tvrdenie	platí?
$f(n) \in O(\sqrt{n} \cdot \sqrt{n})$	
$f(n) \in \Omega(\sqrt{n} \cdot \sqrt{n})$	
$f(n) \in \Theta(\sqrt{n} \cdot \sqrt{n})$	

N - KMP

```
m = n // 20  # kmp preparation
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cur = 0
for i in range(n):
    while cur != 0 and A[i] != B[cur]:
        cur = kmp[cur]
    if A[i] == B[cur]:
        cur += 1
```

$f(n) \approx n$

Tvrdenie	platí?
$f(n) \in O(\sqrt{n} \cdot \sqrt{n})$	✓
$f(n) \in \Omega(\sqrt{n} \cdot \sqrt{n})$	✓
$f(n) \in \Theta(\sqrt{n} \cdot \sqrt{n})$	✓

O

```
nech p = [a1, a2, a3, .... an] | ai je integer
sort(p)
```

$f(n) \in O(\quad)$

O

```
nech p = [a1, a2, a3, .... an] | ai je integer
sort(p)
```

$f(n) \in O(n \log n)$

P

```
nech p = [a1, a2, a3, .... an] | ai je integer
min(p)
```

$f(n) \in O(\ )$

P

```
nech p = [a1, a2, a3, .... an] | ai je integer
min(p)
```

$f(n) \in O(n)$

R

```
i = 0
nech p = [a1, a2, a3, .... an] | ai je integer
if 5 in p:
    i += 1
```

$f(n) \in O(\ )$

R

```
i = 0
nech p = [a1, a2, a3, .... an] | ai je integer
if 5 in p:
    i += 1
```

$f(n) \in O(n)$

S

```
i = 0
nech s = "..." je nejaký reťazec dlhý m znakov
if "x" in s:
    i = s + s
```

$f(n) \in O(\quad)$

S

```
i = 0
nech s = "..." je nejaký reťazec dlhý m znakov
if "x" in s:
    i = s + s
```

$f(n) \in O(m)$

Ponaučenie

1 riadok $\neq$ 1 elementárna operácia

Zložitosti operácií nájdete v dokumentácii. Napr. pre list v Pythone:

Operation	Average Case	Amortized Worst Case
Copy	O(n)	O(n)
Append[1]	O(1)	O(1)
Pop last	O(1)	O(1)
Pop intermediate[2]	O(n)	O(n)
Insert	O(n)	O(n)
Get Item	O(1)	O(1)
Set Item	O(1)	O(1)
Delete Item	O(n)	O(n)
Iteration	O(n)	O(n)
Get Slice	O(k)	O(k)
Del Slice	O(n)	O(n)
Set Slice	O(k+n)	O(k+n)
Extend[1]	O(k)	O(k)
Sort	O(n log n)	O(n log n)
Multiply	O(nk)	O(nk)
x in s	O(n)	
min(s), max(s)	O(n)	
Get Length	O(1)	O(1)

Zoraďte funkcie

$47n^3$ $42n$ $23n^2 + 4n + 3$ $0.0001n^2$ 2^n

3^n $n^{200} \cdot 1.99^n$ n^n $2^{\ln n}$ $n!$

$\ln n$ $\ln\left(n^{100}\right)$ $\ln n!$ $\ln n^n$ $n \cdot \ln n$

$n \cdot (\ln n)^2$ $(n+1)!$

Riešenie

1. $\ln n;\ \ln\left(n^{100}\right)$
2. $42n;\ 2^{\ln n}\,(= n^{\ln 2} = n)$
3. $\ln n!$
4. $n \cdot \ln n;\ \ln n^n$
5. $n \cdot (\ln n)^2$
6. $0.0001n^2;\ 23n^2 + 4n + 3$
7. $47n^3$
8. $n^{200} \cdot 1.99^n;\ 2^n;\ 3^n$
9. $n!$
10. $(n+1)!$
11. n^n