# $k$ - CENTERS

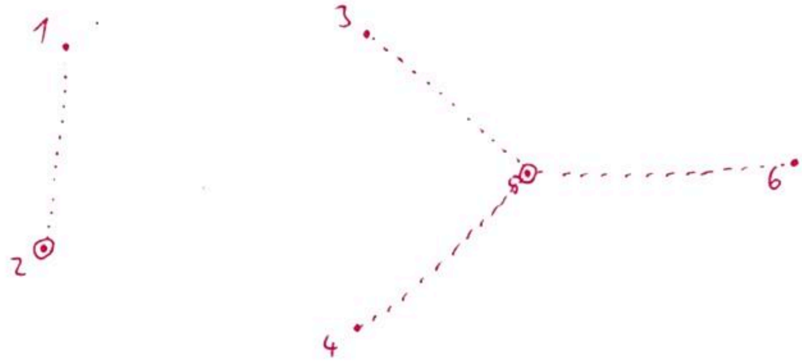
$G = (V, E)$

$COST(e)$ SATISFYING TRIANGLE INEQUALITY

$CONNECT(v, S)$ = COST OF THE CHEAPEST EDGE FROM $v$ TO A VERTEX IN $S \subseteq V$

GOAL: FIND SET $S \subseteq V$, $|S| = k$ WHICH MINIMIZES $\max_{v \in V} (CONNECT(v, S))$

EXAMPLE:




$k = 2 \Rightarrow S = \{5, 2\}$

ALGORITHM (2-APX):

$S = \{1\}$

FOR $i$ IN $2 \dots k$:

$v = \underset{v \in V}{ARGMAX} (CONNECT(v, S))$

$S = S \cup \{v\}$

PROOF:

LET $l = \max_{v \in V} (CONNECT(v, S_{OPT}))$

W.L.O.G. $|1', 1| \leq l$

↳ $|2', 1| \leq l \Rightarrow |1', 2'| \leq |1', 1| + |2', 1| \leq 2l$

↳ $|2', 1| > l \Rightarrow |2', i| \leq l \Rightarrow$ CONTINUE WITH ADDING

$\forall v \in V: |v, i| + |i, i'| \leq 2l$