NEAPPROXIMOVATEĽNOSŤ VŠEOB. TSP

THEOREM (CLRS 35.3):

IF $P \neq NP$, THEN FOR ANY CONST. $\rho \geq 1$, THERE IS NO POLYNOMIAL TIME APPROX. ALG. WITH APPROX. RATIO ρ FOR GENERAL TSP.

$$\rho \geq \max\left(\frac{C}{C^*}, \frac{C^*}{C}\right)$$

PROOF: - BY CONTRADICTION

SUPPOSE $\exists A \in APX$, WITH $\rho \geq 1$

USE A TO SOLVE HAMILTONIAN CYCLE PROBLEM IN POLYNOMIAL TIME

HAM $\in$ NP-COMPLETE

$\Rightarrow P = NP$

LET $G = (V, E)$ BE INSTANCE OF HAMILTONIAN-CYCLE

LET $G' = (V, E')$ BE COMPLETE GRAPH $\left(E' = \{(u,v) : u, v \in V \wedge u \neq v\}\right)$

ASSIGN COST:

$$c(u,v) = \begin{cases} 1 & \text{IF } (u,v) \in E \\ \rho|V| + 1 & \text{ELSE} \end{cases}$$

CONSIDER $TSP(G', c)$

IF G CONTAINS HAM. CYCLE H $\Rightarrow$ G' CONTAINS TOUR OF COST $|V|$

ELSE $\Rightarrow \forall$ TOUR OF G' CONTAINS $e \notin E$

$\hookrightarrow \Rightarrow c(\text{TOUR}) \geq (\rho|V| + 1) + (|V| - 1) = \rho|V| + |V| > \rho|V|$ ($\rho + 1$ FACTOR)

BECAUSE $A \in \rho$-APX

IF G HAS HAM $\Rightarrow A(G', c) \rightarrow H$ WITH COST $|V|$

IF G DOES NOT HAVE HAM $\Rightarrow A(G', c) \not\rightarrow H$ WITH COST $\geq \rho|V| + |V|$ $\square$

GEN. TSP → METRIC TSP

GEN. TSP:

$$D(i,j) \geq 0$$

METRIC TSP:

$$M = \max_{i,j} D(i,j)$$

$$D'(i,j) = D(i,j) + M$$

PROOF THAT IT IS METRIC NOW:

$$D'(i,j) + D'(j,k) = D(i,j) + D(j,k) + 2M \geq 2M \geq D(i,k) + M = D'(i,k)$$

PROOF THAT SOLUTION OF METRIC TSP $\neq$ GEN. TSP:

$$2\text{-APX } A(G', D') \rightarrow \text{TOUR OF COST} \leq 2(c + nM) = 2c + 2nM$$

$$2c + 2nM \underbrace{- nM}_{\text{EACH EDGE WAS } +M} = 2C + nM$$

↑ THIS IS WAY WORSE THAN ρ-APX

2-APX METRIC STEINER TREE

$X = R \cup S$, R is required, S is OPTIONAL

$d: X \times X \to \mathbb{R}_{\geq 0}$

+ TRIANGLE INEQUALITY

FIND TREE $T = (V, E)$, WHERE V IS ANY SET $R \subseteq V \subseteq X$ S.T.

$COST(T) = \sum d(u, v)$ IS MINIMIZED

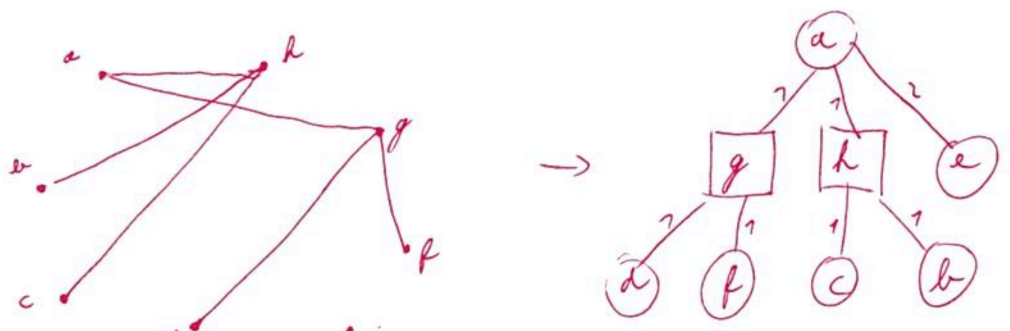
1) COMPLETELY DISREGARD OPTIONAL VERTICES S
2) FIND MIN. SPAN. TREE

PROOF: $(X = (R \cup S), d)$ - INSTANCE OF METRIC STEINER TREE

$T = (V, E)$

$T' = (R, E')$ S.T.

$COST(T') \leq 2\ COST(T)$



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OSTATNÍ 2

$c = 16 \Rightarrow a \to g \to d \to g \to f \to g \to a \to h \to c \to h \to b \to h \to a \to e \Rightarrow$

$\Downarrow$ × OPTIONAL
× USED

$c = 10 \Rightarrow a \Rightarrow d \to f \to c \to b \to e$